

- If you need additional conditions to solve a problem, please write down your assumptions.
- Please put a square box around your final answers for each problem.

Your Name	
Student ID #	
Signature	

Problem	Points	Points Earned
<b>1)</b>	15	
<b>2)</b>	20	
<b>3)</b>	15	
<b>4)</b>	25	
<b>5)</b>	30	
<b>Total</b>	105	

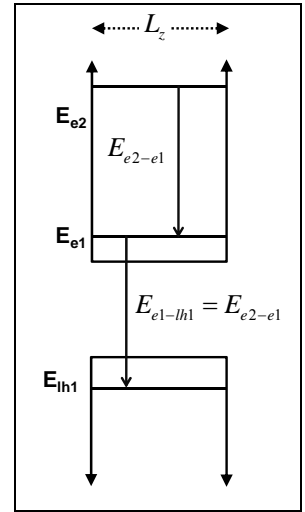
**Table 1:** use the following for all problems unless stated otherwise explicitly:

Optical matrix element:	$M_b^2 = (m_0/6) E_p \approx 6 \times 10^{-49} \left[ \frac{m^2 kg^2}{s^2} \right],$ <p>the value corresponds to <math>E_p = 24</math> eV</p>
Bandgap energy:	$E_g = 1$ eV
Relative dielectric constant:	$\epsilon_r = 9$
Refractive index:	$n_r = 3$
Approximation for $C_0$ in absorption/gain coefficient:	$\alpha_0(\hbar\omega) = C_0 \left  \hat{e} \cdot \vec{P}_{cv} \right ^2 \rho_r(\hbar\omega - E_g)$ $C_0 = \frac{\pi e^2}{n_r c \epsilon_0 m_0^2 \omega} = 10^{10} \cdot \left( \frac{1 \text{ eV}}{\hbar\omega \text{ in eV}} \right) \left[ \frac{m^2}{kg} \right]$
Useful number	$\frac{m_0}{\pi \hbar^2 (1nm)} = 2.6 \times 10^{46} \left[ \frac{s^2}{m^5 kg} \right]$

1. Consider a p-i-n photodetector whose absorption length is  $2 \mu\text{m}$  and area is  $100 \mu\text{m}^2$ . The absorption coefficient is a function of photon wavelength, and can be approximated by  $\alpha(h\nu) = \alpha_0(h\nu - 0.8)^2$ , where  $\alpha_0 = 10^5 \text{cm}^{-1}$ , and  $h\nu$  is photon energy in eV. For all detectors, assume its surface is anti-reflection (AR) coated, and its internal quantum efficiency is 100%. The electron velocity is  $10^7 \text{cm/sec}$ , and the hole velocity is  $10^6 \text{cm/sec}$ . The load resistance is  $50\Omega$ .
  - a) Find the quantum efficiencies of the photodetector at  $1\mu\text{m}$  and  $1.5\mu\text{m}$  wavelengths.
  - b) Find the bandwidth of the photodetector.
  - c) Find the optical powers at which the shot noise is equal to the thermal noise, for both  $1\mu\text{m}$  and  $1.5\mu\text{m}$  wavelengths.
  
2. Consider an APD with a noise figure of 10dB and a gain of 20dB, and p-i-n photodetector. The absorption regions of the APD and the p-i-n are both  $2\mu\text{m}$ , and the absorption coefficient is  $10^4 \text{cm}^{-1}$ . Both of them are connected to load resistors of  $50 \Omega$ . The optical wavelength is  $1.24\mu\text{m}$ . The data bandwidth is 1 GHz.
  - a) Find the optical power at which the signal-to-noise ratios (SNRs) of the APD and the p-i-n are equal.
  - b) For  $\text{SNR} = 1000$ , should one choose APD or p-i-n? Why?
  - c) If the  $k$  parameter of the APD is 0.1, and the electron ionization coefficient is  $10^4 \text{cm}^{-1}$ , what is the thickness of the multiplication region?
  - d) What is the bandwidth of the APD (assuming it is not RC-limited)?
  
3. Consider a quantum well with a width of 10nm, a bandgap of 1eV, an electron effective mass of  $0.1m_0$  and a hole effective mass of  $0.5m_0$ . *For this problem, consider just one hole band with a single hole effective mass.*
  - a) What is the maximum available gain of the quantum well? (You can use the parameters in Table 1 on the cover page).
  - b) In a quantum well laser, the threshold gain should be kept below the maximum available gain of the quantum well. If this quantum well is used as the active media of a quantum well laser with a confinement factor of 1%, what is the minimum length of the laser to keep threshold current low? Assume the laser has cleaved facets with 30% reflectivity and an internal loss of  $10 \text{cm}^{-1}$ .
  - c) If the quantum well laser is *shorter* than the minimum width in (b), what would be the minimum threshold carrier concentration?
  
4. Consider a double heterostructure (DH) laser with an active layer thickness  $d$ . The refractive indices of the core and cladding layers are 5 and 4, respectively. Assume the laser width is  $w$  and length is  $L$ , internal loss is  $\alpha_i$ , mirror loss of  $\alpha_m$ . The gain can be approximated by the linear gain model:  $g(N) = a(N - N_{tr})$ . Assume the carrier lifetime  $\tau$  is constant.

- Find the expression of the confinement factor of the active layer. (Please use the approximate analytical expression rather than the full integral).
- Derive the expression of the threshold current.
- Show that there is an optimum active layer thickness at which the threshold current is lowest.
- Find the expression of the optimum thickness of the active layer.
- Find the expression of the minimum threshold current when active layer thickness is optimized.

5. Consider a *tensile*-strained quantum well with width  $L_z$  such that the electron-to-light hole transition is the lowest energy interband transition, and its energy is equal to the electron intersubband transition, as shown in the Figure. Both transitions can be used as gain media for laser. The  $E_{e2-e1}$  transition is known as quantum cascade laser (QCL), while the  $E_{e1-lh1}$  transition is the normal interband quantum well laser (QWL).



- What are the polarizations of the QCL and QWL?
- Find expression for the *ratio* of the *maximum* optical gain of the QCL and QWL. (Please consider polarization dependent gain).
- For typical semiconductor with  $m_e^* = 0.1m_0$ ,  $m_{lh,z}^* = 0.1m_0$ ,  $m_{lh,t}^* = 0.5m_0$ ,  $E_p = 6$  eV (optical matrix element),  $L_z = 5$  nm, intraband relaxation time  $\tau_{in} = 0.1$  ps, and  $E_{e1-lh1} = E_{e2-e1} = 1$  eV, what is the *ratio* of the maximum optical gain?
- Can you estimate the bandgap energy of the quantum well material using the parameters in Part (c)?
- What is the ratio of the TE/TM gain for the QWL at bandedge?
- What is the ratio of the TE/TM gain for the QWL when the photon energy is 0.1 eV larger than  $E_{e1-lh1}$ ?